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Stem and total tree volume equations, taper and site index, for timber species in the temperate ecosystem of Durango, Mexico

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ABSTRACT

Objective: To generate stem volume (SV), total tree volume (TTV), taper and site index (IS) models for *Pinus arizonica*, *Pinus durangensis*, *Pinus engelmanni*, *Pinus leiophylla*, *Pinus teocote*, *Pinus strobiformis*, *Pinus lumholtzii* and *Quercus sideroxyla*, in productive landscapes of a temperate ecosystem in northwestern Durango, Mexico.
Methodology: Data were collected from 1280 trees (298 with trunk analysis). Based on the dendrometric type, the volume was obtained and estimated the SV, TTV and VC (branches ≥5 cm volume). Using a xylometer, the volume of branches and twigs <5 cm was estimated. For TTV and SV, the Schumacher-Hall model was fitted with a ponderaded regression to correct for heteroscedasticity. The Biging function was used for the stem profile, fitted with nonlinear generalized least squares to correct for autocorrelation. The algebraic difference method with Chapman-Richards base equation was used to generate the site index equations.
Results: A biometric system has been developed with 16 equations to estimate SV and TTV, 8 equations for stem profile, and 14 equations to form polymorphic curves and qualify the IS.
Limitations: Its use elsewhere, requires a validation process.
Conclusions: The results show that the models were unbiased and with significant parameters, so their use is recommended in the estimation of the SV, TTV, stem profile and estimation of the IS in the region of Santiago Papasquiaro, Durango, Mexico.

Keywords: stem volume, total tree volume, taper, site index.

INTRODUCTION

Accurate, efficient, and up-to-date quantitative information on trees and forest stands is essential for the assessment and development of sustainable forest management practices within ecosystems (Fukumoto *et al.*, 2021; Raj Aryal *et al.*, 2023; Ma *et al.*,

2024). Such data are critical for informed decision-making in areas including ecosystem management, climate change assessment, productivity estimation, ecosystem services provisioning, and carbon cycle studies (Ma *et al.*, 2024). Recent research trends have shifted from stand-level yield prediction to individual-tree level forecasting (Fukumoto *et al.*, 2021). Forest modeling approaches such as tree volume, taper or stem profile, and site quality models are fundamental tools for generating this information (Raj Aryal *et al.*, 2023; Roitsch *et al.*, 2023). Hossain (2025) highlighted the critical need for species- and site-specific modeling frameworks. The estimation of merchantable stem volume and total tree volume is a cornerstone in the management of temperate forests (Flores *et al.*, 2021; Monárrez-González *et al.*, 2024). These estimates are commonly obtained through equations including Schumacher-Hall, Combined Variable, Dwight, Korsun, Australian, Modified Meyer, Naslund, Takata, Comprehensive, Logarithmic, Logarithmic without intercept, Thornber, Berkhout, Honer, and Wenk, among others (Clutter *et al.*, 1983; Romahn *et al.*, 1994; Prodan *et al.*, 1997). Due to its precision and simplicity, the Schumacher-Hall model remains one of the most widely used (Corral & Návar-Cháidez, 2009; Vargas-Larreta *et al.*, 2018). By incorporating the combined variable (D×H, where D=diameter at breast height and H=total height), it enables improved estimations of both volume and biomass (Pienaar *et al.*, 2024). Tree volume can be estimated either in components or as a whole, the latter including stem, branches, and twigs (Vargas-Larreta *et al.*, 2018). The stem volume specifically refers to the amount of wood contained within the cylindrical portion of the trunk, excluding branches and crown. Taper functions, or stem profile models, are employed to estimate the volume of specific stem sections by predicting diameter measurements at any given point along the stem (Burkhart & Tomé, 2012b). Numerous taper equations have been developed, including those by Bruce *et al.* (1968), Demaerschalk (1972), Ormerod (1973), Max & Burkhart (1976), Cao *et al.* (1980), Hilt (1980), Biging (1984), Clark *et al.* (1991), Rentería & Maldonado (1998), Kozak (1998), Fang *et al.* (2000), Bi (2000), Sharma & Oderwald (2001), as well as Cielito 1, 2, and 3 (Rentería *et al.*, 2006; Cobos *et al.*, 2023). Among these, the equations developed by Biging (1984) and Fang *et al.* (2000) have proven particularly accurate for describing stem profiles (Corral & Návar-Cháidez, 2009; Marín *et al.*, 2009). Notably, Biging's model offers the advantage of using only two parameters. The National Forestry Commission and other Mexican institutions jointly undertook a project to develop a biometric system for forest management planning across 13 states in Mexico, resulting in the formulation of 6,414 equations for volume estimation, taper modeling, and site quality assessment. When estimating total tree volume, this biometric system only accounted for branches and twigs with diameters greater than 5 cm. Moreover, it did not produce separate equations for stem and total tree volume, though such estimates were derived using additive models. In the context of traditional forest management and commercial practices in Mexico, the use of separate or differentiated equations for stem volume, total tree volume, and commercial volume taper is often more practical (Monárrez-González *et al.*, 2024). Site index curves are essential tools for assessing forest productivity potential. The site index is defined as the average height of dominant stand trees at a reference age (Burkhart & Tomé, 2012a). This relationship yields a curve used to quantify site productivity (Riofrío *et al.*, 2023; Stefanello *et al.*, 2024). Site index estimation involves developing mathematical models that infer the growth trajectories of individual trees or stands. Commonly employed models include Weibull, Schumacher, Chapman-Richards, Hossfeld IV, and Korf (Stefanello *et al.*, 2024). In this study, the Chapman-Richards model was selected as the base equation. The aim of this study was to develop stem volume, total tree volume, taper, and site index equations for *Pinus arizonica* Engelm., *Pinus durangensis* Mart., *Pinus engelmanni* Carr., *Pinus leiophylla* Schl. & Cham., *Pinus teocote* Schiede ex Schltdl., *Pinus strobiformis* Engelm., *Pinus lumholtzii* Rubins & Ferns, and *Quercus sideroxyla* Bonpl., within productive landscapes of a temperate ecosystem in northwestern Durango, Mexico.

MATERIALS AND METHODS

The study was conducted in the region of Santiago Papasquiaro, Durango, Mexico, within Forest Management Unit 1005, located at geographic coordinates 25° 03' N latitude and 105° 29' W longitude (Figure 1). The prevailing climate is temperate sub-humid with summer rainfall, characterized by a mean annual temperature of 17.7 °C and an average annual precipitation of 600 millimeters (INAFED, 2009).

Through interactions with technical service providers and forest producers, the most commercially significant timber species were identified: *Pinus arizonica* Engelm. (Species 1), *Pinus durangensis* Mart. (Species 2), *Pinus engelmanni* Carr. (Species 3), *Pinus leiophylla* Schl. & Cham. (Species 4), *Pinus teocote* Schiede ex Schltdl. (Species 5), *Pinus strobiformis* Engelm. (Species 6), *Pinus lumholtzii* Rubins & Ferns (Species 8), and *Quercus sideroxyla* Bonpl. (Species 9).

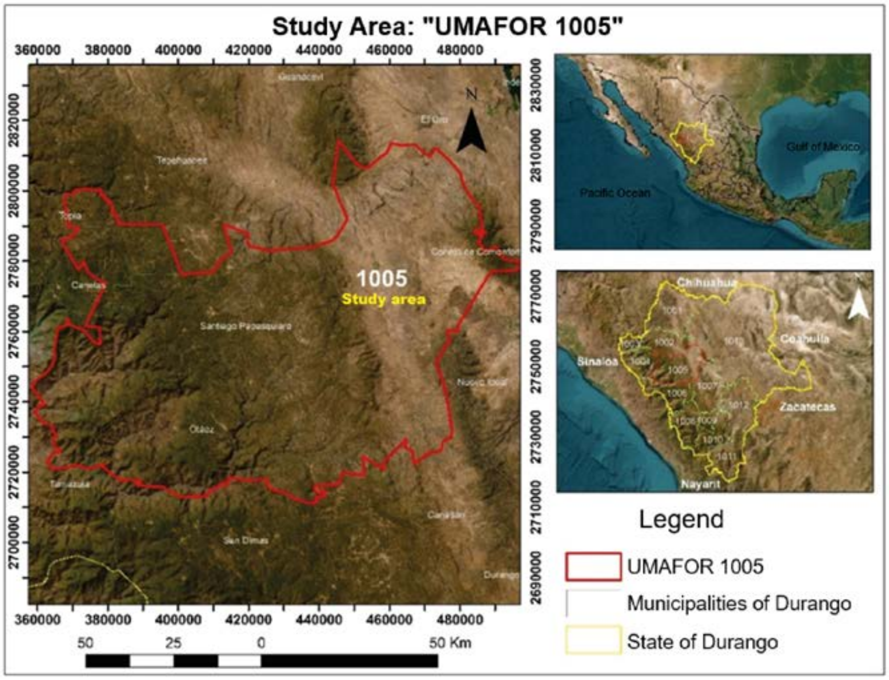

Figure 1. Location of the study area.

Data collection and measured variables

Trees were felled and measured by diameter class (DC) and by species. A targeted sampling approach was employed to ensure representation across all site quality levels and diameter classes (increments of 5 cm), as well as height variation. For volume measurements, each tree was divided into stem and crown components. The stem was cut at 0.3 m above ground level. Measurements were taken at two 0.30 m sections above the stump, up to the height corresponding to diameter at breast height (DBH, 1.3 meters). Thereafter, stem measurements continued at 2-meter intervals up to the tree tip. The final section length did not exceed 1.3 meters. The crown was subdivided into: (a) branches with a minimum diameter ≥ 5 cm and lengths ≤ 2 m, and (b) branches and twigs with minimum diameters <5 cm. Branches and twigs with diameters under 5 cm were weighed using a 40-kg commercial scale. Three subsamples (from the upper, middle, and lower crown sections) were extracted, weighed, and their volume determined using a calibrated water drum (xylometer). Measurement instruments included a diameter tape, a 30-meter measuring tape, and a gasoline-powered chainsaw for cutting. The variables recorded were: diameter at breast height (DBH, cm), total height (H, m), stump height, bark-included diameter for each section, section length, bark thickness, bark-included diameter for each branch section, branch section length for diameters ≥ 5 cm, total crown weight, subsample weight, and branch volume per subsample. Additionally, stem analysis was conducted on 298 Pinus specimens to support growth prediction and site index estimation, following the methodology proposed by Klepac (1983).

Volume calculation

The volume of each section was estimated using the appropriate dendrometric method (Table 1). An exception was made for branches and twigs with diameters <5 cm, whose volume was estimated via weight-to-volume ratio. These data were then used to calibrate the allometric combined-variable equation (Prodan *et al.*, 1997):

$$Vr = \beta_0 + DN^2 AT^{\beta_1} \tag{1}$$

Where: Vr=Volume of branches and twigs (m^3) less than 5 cm, DN=Diameter at breast height (cm), H=Total height (m), β_i=Coefficients.

The total volume of the tree with bark (VTA) was estimated by adding the volume of the stump + stem volume + tip volume + crown volume of branches with diameters greater than or equal to 5 cm + volume of branches and twigs with diameters less than 5 cm. The volume of the stem with bark (VF) was estimated by adding the volume of the stump + stem volume + tip volume.

Model fitting, stem volume and total tree volume

The estimation of stem volume (SV) and total tree volume (TWV) was performed using the Schumacher-Hall equation (Schumacher and Hall, 1933):

Table 1. Equations used for the cubication of felled trees.

Stem	Dendrometric Type	Formula
Tip	Apollonian paraboloid	$V=\frac{Ao}{2}$
Sections	Apolonian paraboloid truncation	$V=\left(\frac{Ao+A1}{2}\right)L$
Stump	Truncated neiloid	$V=\frac{L}{4}\left(S_0+S_1+\sqrt[3]{S_0S_1}\left(\sqrt[3]{S_0}+\sqrt[3]{S_1}\right)\right)$
Branches	Apolonian paraboloid truncation	$V=\left(\frac{Ao+A1}{2}\right)L$

V=Volume (m^3); Ao=Area of the smaller section (m^2); $A1$=Area of the larger section (m^2); S_0 and S_1=Larger and smaller area; L=Length (m). Source: Romahn *et al.* (1994).

$$V=\beta_0 DN^{\beta_1} AT^{\beta_2} \tag{2}$$

Where: DN=Normal diameter with bark (cm), H=Total height (m), β_i=Coefficients, V=Volume (m^3).

The equations were fitted using the NLIN procedure in SAS© (SAS Institute, 2015). The assumptions of error independence and homoscedasticity with zero mean and constant variance were verified, along with tests for multicollinearity, autocorrelation, and heteroscedasticity. To enhance outlier detection, a non-parametric quadratic local fit was applied using LOESS local regression (Cleveland, 1993), assuming normally distributed errors. A smoothing parameter of 0.3 was used for each species.

Residuals versus predicted values indicated the presence of heteroscedasticity across all species. Consequently, a weighted regression was employed, assigning weights equal to the inverse of the variance of each observation. Model performance was evaluated using the standard error of the estimate (STDE) and the adjusted coefficient of determination (R^{2}adj).

Model fitting, taper

The Biging equation was used to describe the stem profile (Biging, 1984). This expression is based on the integral form of the Bertalanffy-Richards equation with two parameters (Biging, 1984):

$$d=D\left(\beta_1+\beta_2\ln\left(1-q^{\frac{1}{m}}\left(1-\exp^{\frac{-\beta_1}{\beta_2}}\right)\right)\right) \tag{3}$$

Where: D=Final diameter (cm), d=Diameter at different heights along the stem (cm), β_i=Regression coefficients, q=(h/H), h=Initial height, ln=Base of natural logarithms, m=Constant value of 3 (Corral *et al.*, 1999).

Since correlation was detected among the observations violating the assumption of error independence nonlinear generalized least squares (NGLS) were applied, expanding the error term using a continuous autoregressive model of order x [CAR(x)]. This error structure allows for the application of models to irregularly spaced or unbalanced data. In a second-order continuous autoregressive model [CAR(2)], the error structure was simultaneously fitted alongside the mean function for each equation using the MODEL procedure from the SAS/ETS statistical package (SAS© Institute, 2015), which supports dynamic updating of residuals.

Site index equations

To generate the site index curve family, the algebraic difference approach was employed. The initial step involved deriving the Chapman -Richards height -age equation in its algebraic difference form. To determine the type of curve family anamorphic or polymorphic the generating parameter was identified. The equation was then solved with respect to that parameter using two consecutive measurements of dominant height (H_1 and H_2) and age (E_1 and E_2). By equating both expressions and solving for the second height measurement (H_2), the resulting formulation was fitted to the dataset (Table 2).

The base age used corresponded to the age at which trees typically reach a commercial bark-included diameter of 25 to 30 cm.

Accepting the polymorphism of tree growth, developing in different site qualities, the parameter β_2 was considered to generate the polymorphic curves for the Chapman-Richards model (Table 3). The equations used in each method were adjusted using the Statistical Analysis System (SAS) package and the NLIN procedure, which includes the DUD method (which does not use derivatives), preliminary parameter values were

Table 2. Fitted equations using the algebraic difference method to derive anamorphic and polymorphic site index curves.

Equation	Curves for anamorphic curves	Equation for polymorphic curves
Chapman-Richards	$H_2 = H_1\left(\frac{1-e^{-\beta_1 E2}}{1-e^{-\beta_1 E1}}\right)^{\beta_2}$	$H_2 = \beta_0\left(\frac{H_1}{\beta_0}\right)^{\left(\frac{\ln\left(1-e^{-\beta_1 E2}\right)}{\ln\left(1-e^{-\beta_1 E1}\right)}\right)}$

H_2=Dominant height at age 2 (years); H_1=Dominant height at age 1 (years); E_2=Age 2 (years); E_1=Age 1 (years); e=Base of the natural logarithm; β_i=Parameters to be estimated. Source: García, 1998.

Table 3. Equations to generate the family of polymorphic curves and to qualify their site index, using the algebraic difference method.

Equation	To generate polymorphic curves	To rate the Site Index
Chapman-Richards	$H = \beta_0\left(\frac{IS}{\beta_0}\right)^{\left(\frac{\ln\left(1-e^{-\beta_1 E}\right)}{\ln\left(1-e^{-\beta_1 EB}\right)}\right)}$	$IS = \beta_0\left(\frac{H}{\beta_0}\right)^{\left(\frac{\ln\left(1-e^{-\beta_1 EB}\right)}{\ln\left(1-e^{-\beta_1 E}\right)}\right)}$

H=Dominant height (years); SI=Site index (years); E=Age (years); BA=Base age (years); e=Base of the natural logarithm; β_i=Parameters to be estimated. Source: García, 1998.

obtained. These values were subsequently used in the Marquardt method. Microsoft Excel (Office XP) was used to create and present the final graphs.

RESULTS AND DISCUSSION

The database consists of 1280 trees; stem analysis was performed on 298 of them. The statistics describing the data are shown in Table 4 and Figure 2.

Volume

The volume of branches and twigs smaller than 5 cm was estimated at the genus level. A database of 40 Pinus trees was used to fit the model. A good fit and significant parameters were observed (Table 5).

For *Quercus sideroxyla*, only the total tree volume including branches with diameters equal to or greater than 5 cm was considered. Once the individual tree volume was calculated by species, the Schumacher-Hall model was fitted for both stem volume (SV) and total tree volume (TTV). Across all species, the model exhibited low bias and statistically significant

Table 4. Descriptive statistics of normal diameter, total height and volume of the sample used by species.

Species	Number of trees	Normal diameter (cm)		Total height (m)		Volume (m^3)	
		Min.	Max.	Min.	Max.	Min.	Max.
Pinus arizonica Engelm. (sp. 1)	332	4	56	2.74	33	0.00893	4.53831
Pinus durangensis Mart. (sp. 2)	281	4	61	2.78	36.02	0.00547	5.19903
Pinus engelmanni Carr. (sp. 3)	74	4	58	2.38	25.7	0.01217	5.08743
Pinus leiophylla Schl. & Cham. (sp. 4)	151	3	66	2.63	30	0.00809	5.68319
Pinus teocote Schiede ex Schltdl. (sp. 5)	168	3	61	2.84	35.31	0.00594	5.63632
Pinus strobiformis Engelm. (sp. 6)	131	5	64	3.2	29.98	0.01283	5.11872
Pinus lumholtzii Rubins & Ferns (sp. 7)	14	7	43	3.85	18	0.02548	1.84704
Quercus sideroxyla Bonpl. (sp. 8)	129	10	56.2	6	26.8	0.02660	3.1132
Total	1280						

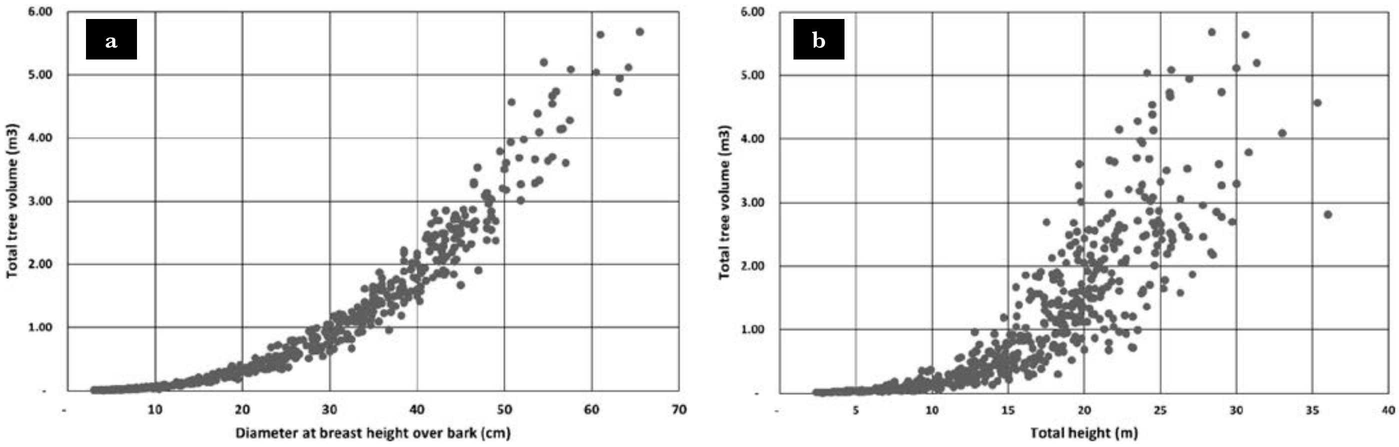

Figure 2. Graph of total tree volume data: (a) versus diameter at breast height and (b) versus total height, in Pinus species.

Table 5. Statistical parameters of the equation to estimate volume in branches and twigs less than 5 cm of Pinus.

Genus	Model	β_0	β_1	Aprox. Pr>F	R^{2}adj
Pinus	$Vr = 0.0000062683DN^2H^{1.028396}$	0.000002839	01.028396	<.0001	0.834

DN=Diameter at breast height with bark (cm); H=Total height (m); V_{r}=Volume of twigs (m^3); β_i=Estimator coefficients; R^{2}adj=Adjusted coefficient of determination.

parameters. Notably, the *Quercus sideroxyla* model for SV and TTV recorded the lowest coefficient of determination (Tables 6 and 7).

In the Table 8. Equations for estimating stem volume (SV) and total tree volume (TTV).

Various studies, consistent with the findings of this research, have demonstrated that the Schumacher-Hall model yields the highest coefficients of determination and the lowest standard errors, making it a reliable and accurate tool for estimating stem volume (SV) and total tree volume (TTV). Notable examples include: Corral and Návar (2009), who evaluated seven models for estimating stem volume in *Pinus cooperi*, *Pinus durangensis*, *Pinus engelmannii*, *Pinus leiophylla*, and *Pinus herrerae* in Durango; Tapia and Návar (2011), who fitted and validated eight volume equations for *Pinus pseudostrobus*; Ramos-Uvilla *et al.* (2014), who developed volume equations for *Pinus lawsonii* and *Pinus oocarpa*, assessing five models; Ramírez-Martínez *et al.* (2018), who fitted models to estimate total volume in *Pinus*

Table 6. Statistical parameters of the Schumacher-Hall model for estimating stem volume.

Species	β_0	β_1	β_2	STDEβ_0	STDEβ_1	STDEβ_2	R^{2}adj
Pinus durangensis Mart.	0.0001203	2.053449	0.62453	0.0000341	0.1007263	0.1116093	0.99
Pinus arizonica Engelm.	0.0001605	2.031317	0.55840	0.0000158	0.0585563	0.0694241	0.98
Pinus leiophylla Schl. & Cham.	0.0001540	1.843831	0.77769	0.0000155	0.0462991	0.0594794	0.98
Pinus teocote Schiede ex Schltdl.	0.0001341	1.819946	0.87184	0.0000117	0.0435401	0.0545277	0.98
Pinus engelmanni Carr.	0.0001068	2.036591	0.67959	0.0000183	0.0655600	0.0947218	0.98
Pinus lumholtzii Rubins & Ferns	0.0000632	1.697588	1.29632	0.0000254	0.2256554	0.2634072	0.99
Pinus strobiformis Engelm.	0.0001460	1.815767	0.80372	0.0000138	0.0439247	0.0605563	0.99
Quercus sideroxyla Bonpl.	0.0000521	2.062287	0.80431	0.0000051	0.0407067	0.0594119	0.97

β_i=Estimator parameters; STD=Standard error of the estimator; R^{2}adj=Adjusted coefficient of determination.

Table 7. Statistical parameters of the Schumacher-Hall model for estimating total tree volume.

Species	β_0	β_1	β_2	STDEβ_0	STDEβ_1	STDEβ_2	R^{2}adj
Pinus durangensis Mart.	0.0001090	2.098616	0.66262	0.0000106	0.0363408	0.0407430	0.99
Pinus arizonica Engelm.	0.0001452	2.105864	0.56526	0.0000126	0.0443777	0.0538442	0.98
Pinus leiophylla Schl. & Cham.	0.0001441	1.936104	0.75263	0.0000133	0.0421186	0.0540458	0.99
Pinus teocote Schiede ex Schltdl.	0.0001237	1.910064	0.85239	0.0000105	0.0411985	0.0511956	0.99
Pinus engelmanni Carr.	0.0000962	2.179124	0.61544	0.0000160	0.0635352	0.0910050	0.98
Pinus lumholtzii Rubins & Ferns	0.0000667	1.776869	1.23615	0.0000232	0.1984066	0.2293798	0.99
Pinus strobiformis Engelm.	0.0001341	1.873057	0.82641	0.0000113	0.0377439	0.0523821	0.99
***Quercus sideroxyla* Bonpl.	0.0000423	2.302564	0.63186	6.84E-06	0.0608740	0.0814590	0.96

β_i=Estimator parameters; STDβ_i=Standard error of the estimator; R^{2}adj=Adjusted coefficient of determination. **TTV includes only branches with diameters less than or equal to 5 cm.

Table 8. Equations for estimating the total volume of a tree with bark.

Species	Stem volume	Total tree volume
(sp. 1)	$SV=0.000120379\ DN^{2.053449}\ H^{0.624537}$	$TTV=0.000109831\ DN^{2.098617}\ H^{0.6626}$
(sp. 2)	$SV=0.000160599\ DN^{2.031317}\ H^{0.558404}$	$TTV=0.000145271\ DN^{2.105864}\ H^{0.5652}$
(sp. 3)	$SV=0.000154074\ DN^{1.843831}\ H^{0.777698}$	$TTV=0.000144147\ DN^{1.936105}\ H^{0.7521}$
(sp. 4)	$SV=0.000134149\ DN^{1.819946}\ H^{0.871845}$	$TTV=0.000123698\ DN^{1.910064}\ H^{0.85239}$
(sp. 5)	$SV=0.000106881\ DN^{2.036591}\ H^{0.679597}$	$TTV=0.00009620\ DN^{2.179124}\ H^{0.615442}$
(sp. 6)	$SV=0.00014609\ DN^{1.815767}\ H^{0.803722}$	$TTV=0.00013413\ DN^{1.873058}\ H^{0.826412}$
(sp. 7)	$SV=0.000063252\ DN^{1.697588}\ H^{1.296322}$	$TTV=0.00006674\ DN^{1.77687}\ H^{1.23615}$
**(sp. 8)	$SV=0.00005219\ DN^{2.0622879}\ H^{0.8043177}$	$TTV=0.000042372\ DN^{2.3025652}\ H^{0.63186}$

SV=Stem volume (m^3); *TTV*=Total tree volume (m^3); *DN*=Diameter at breast height (cm); *H*=Total height (m). ***TTV* includes only branches with diameters less than or equal to 5 cm.

ayacahuite; and finally, Vargas-Larreta *et al.* (2018) and Rodríguez-Flores *et al.* (2019), who applied the Schumacher-Hall model to estimate volume components of several species or species groups within the temperate forests of northwestern Mexico, concluding that the model provided a suitable fit to the data.

Taper modeling

Table 9 presents the estimated parameters and standard errors for the Biging taper model across all evaluated species. The root mean square error (RMSE) and the coefficient of determination for nonlinear regression (R^2) were examined. While certain limitations have been noted regarding the use of adjusted R^2 in nonlinear regression, the general utility of a global measure of model goodness-of-fit appears to outweigh these drawbacks (Ryan, 1997).

Table 10 shows the taper equations with the Biging model.

Based on the statistical indicators, the equations adequately describe the stem profile for each species. Similar to the present study, several investigations have confirmed that the Biging (1984) model accurately characterizes stem taper as a function of diameter at breast height. Notable examples include: Rentería and Maldonado (1998), who evaluated

Table 9. Statistical parameters of the Biging (1984) model for taper.

Species	β_0	β_1	STDEβ_0	STDEβ_1	RMSE	R^2adj
Pinus durangensis Mart.	1.250738	0.376501	0.006057727	0.008439525	3.37112	0.949192
Pinus arizonica Engelm.	1.248987	0.353533	0.004969113	0.005255968	2.450573	0.974216
Pinus leiophylla Schl. & Cham.	1.241395	0.403417	0.005101158	0.00588159	2.20776	0.981302
Pinus teocote Schiede ex Schltdl.	1.203131	0.409441	0.005864491	0.011032187	3.297952	0.958739
Pinus engelmanni Carr.	1.221884	0.376776	0.0069588	0.007789118	2.526474	0.97681
Pinus lumholtzii Rubins & Ferns	1.212478	0.411632	0.016581769	0.021741142	1.8038004	0.986231
Pinus strobiformis Engelm.	1.221699	0.373184	0.008539492	0.011131668	4.231821	0.935971
Quercus sideroxyla Bonpl.	1.267633	0.467729	0.005768442	0.007704512	2.108448	0.982162

β_i=Estimator parameters; STDβ_i=Standard error of the estimator; RMSE=Root mean square error; R^2adj=Adjusted coefficient of determination.

Table 10. Taper equations with Biging (1984).

Species	Tapering equation, Biging (1984)
Pinus durangensis Mart.	$d=D(1.250738+0.376501 \ln (1-(1-e^{1.250738/0.376501})\, q^{1/m}))$
Pinus arizonica Engelm.	$d=D(1.248987+0.353533 \ln (1-(1-e^{1.248987/0.353533})\, q^{1/m}))$
Pinus leiophylla Schl. & Cham.	$d=D(1.241395+0.403417 \ln (1-(1-e^{1.241395/0.403417})\, q^{1/m}))$
Pinus teocote Schiede ex Schltdl.	$d=D(1.203131+0.409441 \ln (1-(1-e^{1.203131/0.409441})\, q^{1/m}))$
Pinus engelmanni Carr.	$d=D(1.221884+0.376776 \ln (1-(1-e^{1.221884/0.376776})\, q^{1/m}))$
Pinus lumholtzii Rubins & Ferns	$d=D(1.212478+0.411632 \ln (1-(1-e^{1.212478/0.411632})\, q^{1/m}))$
Pinus strobiformis Engelm.	$d=D(1.221699+0.373184 \ln (1-(1-e^{1.221699/0.373184})\, q^{1/m}))$
Quercus sideroxyla Bonpl.	$d=D(1.267633+0.467729 \ln (1-(1-e^{1.267633/0.467729})\, q^{1/m}))$

$m=3$; $q=(h/H)$; h and d=Initial height and diameter; D and H=Final diameter and height.

ten taper equations for *Pinus cooperi* in Durango; Corral *et al.* (1999), who validated six taper models for *Pinus cooperi*, *Pinus durangensis*, *Pinus engelmannii*, *Pinus leiophylla*, and *Pinus herrerae* in the El Salto forest region of Durango; Marín *et al.* (2019), who compared taper models for *Pinus arizonica* in southwestern Chihuahua; and Corral and Návar-Cháidez (2009), who concluded that the Biging model provided an excellent fit for predicting tree diameter profiles in *Pinus cooperi*, *Pinus durangensis*, *Pinus engelmannii*, *Pinus leiophylla*, and *Pinus herrerae* in Durango.

Site Index

Site index curves were developed using the algebraic difference method, with a base age of 68 years corresponding approximately to the age at which trees reach commercial bark-included diameter (25 -30 cm). Table 11 presents the fit statistics used to construct the height growth curves for the Pinus species.

Table 11. Statistical parameters of the Chapman-Richards model for height growth curves.

Species	SSE	MSE	Pr>F	R^2adj	Parameter	STDβ_i
Pinus durangensis Mart.	1950.8	0.02899	<.0001	0.994	$\beta_1=31.7443$ $\beta_2=0.0166$	0.7981 0.0010
Pinus arizonica Engelm.	2049.4	0.02319	<.0001	0.995	$\beta_1=37.9632$ $\beta_2=0.0138$	0.9216 0.0006
Pinus leiophylla Schl. & Cham.	1293.8	0.02597	<.0001	0.994	$\beta_1=31.4035$ $\beta_2=0.0152$	1.0862 0.0001
Pinus teocote Schiede ex Schltdl.	1091.1	0.02063	<.0001	0.993	$\beta_1=30.7626$ $\beta_2=0.0156$	1.1824 0.0012
Pinus engelmanni Carr.	549.8	0.02543	<.0001	0.980	$\beta_1=23.1666$ $\beta_2=0.0347$	1.5095 0.0064
Pinus strobiformis Engelm.	248.0	0.01857	<.0001	0.991	$\beta_1=36.4443$ $\beta_2=0.0157$	4.8492 0.0039
Pinus sp.	5953.8	0.02063	<.0001	0.994	$\beta_1=36.0836$ $\beta_2=0.0138$	0.5943 0.0004

β_i=Estimator parameters; SSE=Sum of squared errors; MSE=Mean squared error; Pr>F=Probability of obtaining the F-value; R^2adj=Adjusted coefficient of determination; STDβ_i=Standard error of the estimator.

Accepting the polymorphism of tree growth as true, site index curves are generated, example in Figure 3.

Table 12 shows the equations for generating the polymorphic curves and calculating the site index.

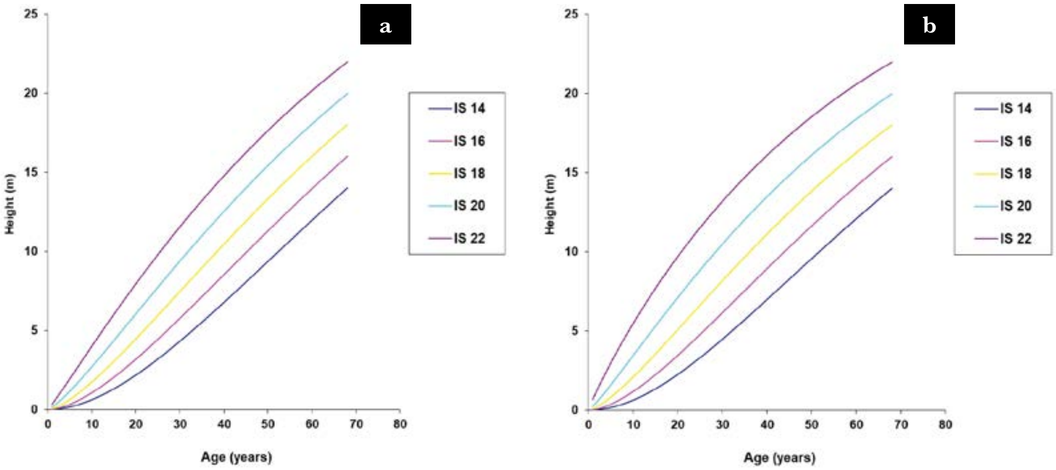

Figure 3. Polymorphic site index curves for *Pinus arizonica* (a) and *Pinus teocote* (b) at a base age of 68 years. IS=site index (m).

Table 12. Equations for generating polymorphic curves and qualifying site index.

Species	Polymorphic curves	Site Index rating
Pinus durangensis Mart.	$H=31.7444\left(1-e^{-0.0166E}\right)^{\left(\frac{\ln\frac{IS}{31.7444}}{\ln\left(1-e^{-0.0166EB}\right)}\right)}$	$IS=31.744\left(1-e^{-0.0166EB}\right)^{\frac{\ln\frac{H}{31.7444}}{\ln\left(1-e^{-0.0166E}\right)}}$
Pinus arizonica Engelm.	$H=37.9633\left(1-e^{-0.0138E}\right)^{\left(\frac{\ln\frac{IS}{37.9633}}{\ln\left(1-e^{-0.0138EB}\right)}\right)}$	$IS=37.963\left(1-e^{-0.0138EB}\right)^{\frac{\ln\frac{H}{37.9633}}{\ln\left(1-e^{-0.0138E}\right)}}$
Pinus leiophylla Schl. & Cham.	$H=33.5262\left(1-e^{-0.0138E}\right)^{\left(\frac{\ln\frac{IS}{33.5262}}{\ln\left(1-e^{-0.0138EB}\right)}\right)}$	$IS=33.526\left(1-e^{-0.0138EB}\right)^{\frac{\ln\frac{H}{33.5262}}{\ln\left(1-e^{-0.0138E}\right)}}$
Pinus teocote Schiede ex Schltdl.	$H=30.7626\left(1-e^{-0.0156E}\right)^{\left(\frac{\ln\frac{IS}{30.7626}}{\ln\left(1-e^{-0.0156EB}\right)}\right)}$	$IS=30.762\left(1-e^{-0.0156EB}\right)^{\frac{\ln\frac{H}{30.7626}}{\ln\left(1-e^{-0.0156E}\right)}}$
Pinus engelmanni Carr.	$H=26.3205\left(1-e^{-0.0279E}\right)^{\left(\frac{\ln\frac{IS}{26.3205}}{\ln\left(1-e^{-0.0279EB}\right)}\right)}$	$IS=26.320\left(1-e^{-0.0279EB}\right)^{\frac{\ln\frac{H}{26.3205}}{\ln\left(1-e^{-0.0279E}\right)}}$
Pinus strobiformis Engelm.	$H=36.4444\left(1-e^{-0.0157E}\right)^{\left(\frac{\ln\frac{IS}{36.4444}}{\ln\left(1-e^{-0.0157EB}\right)}\right)}$	$IS=36.444\left(1-e^{-0.0157EB}\right)^{\frac{\ln\frac{H}{36.4444}}{\ln\left(1-e^{-0.0157E}\right)}}$
Pinus sp.	$H=36.0836\left(1-e^{-0.0138E}\right)^{\left(\frac{\ln\frac{IS}{36.0836}}{\ln\left(1-e^{-0.0138EB}\right)}\right)}$	$IS=36.083\left(1-e^{-0.0138EB}\right)^{\frac{\ln\frac{H}{36.0836}}{\ln\left(1-e^{-0.0138E}\right)}}$

H=dominant height (m); IS=Site index (m); EB=Base age (68 years) and E=Age (years).

CONCLUSIONS

The species-specific fitted equations provide reliable and accurate estimates for determining stem volume, total tree volume, and stem profile for the most commercially important timber species: *Pinus strobiformis* Engelm., *Pinus arizonica* Engelm., *Pinus durangensis* Mart., *Pinus engelmanni* Carr., *Pinus leiophylla* Schl. & Cham., *Pinus lumholtzii* Rubins & Ferns, *Pinus teocote* Schiede ex Schltdl., and *Quercus sideroxyla* Bonpl., within the Santiago Papasquiaro region, Durango, Mexico. Due to its precision and simplicity, the authors recommend the Schumacher-Hall equation for estimating both stem and total tree volume, and the Biging equation for modeling the stem taper profile in timber species. Using the algebraic difference method and the Chapman-Richards base equation, polymorphic curve families were developed to assess site index, with a base age of 68 years. The results demonstrated unbiased models with statistically significant parameters, supporting their application in the Santiago Papasquiaro region of Durango, Mexico.

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