Colegio de Postgraduados

A mixed-effects height-diameter model for *Pinus douglasiana* Martínez in temperate forests of Jalisco, Mexico

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ABSTRACT

Objective: This study aims to develop a generalized mixed-effects model for predicting tree-height of *Pinus douglasiana* in the natural forests of Jalisco, Mexico.

Design/methodology/approach: For this study, we utilized 2,921 pairs of tree-height measurements collected from 65 permanent plots (each 50×50 m) in the study area. For each plot, we estimated the tree density as the number of trees per hectare (N, ha^{-1}), the basal area per hectare (G, m^2 ha^{-1}) and the quadratic mean diameter (dg, cm). Subsequently, we tested several models from the literature and developed a generalized mixed-effects model for tree height prediction.

Results: The Gompertz base model outperformed the other local models, achieving an R^2 of 0.75 and an RMSE of 3.13. Including the stand variables (N, G, and dg) and incorporating random effects significantly improved the model fit, resulting in an R^2 of 0.89 and an RMSE of 2.09 m. Calibration and validation steps revealed that selecting the three thickest trees is effective for estimating random effects in new plots or stands, with the RMSE and R^2 being 2.59 and 0.83, respectively.

Limitations on study/implications: The present model can be applied in areas where *P. douglasiana* is naturally distributed. However, it is advisable to calibrate the model using a sub-sample to achieve more accurate predictions of tree heights in new plots or areas.

Findings/conclusions: The Gompertz function was used as the base function to develop the final generalized mixed-effects model for predicting the height of *Pinus douglasiana*. The inclusion of stand variables (N, G, and dg) along with random effects improved the model fit. This model represents a new and more accurate tool for predicting the height of *P. douglasiana* in areas where it is naturally distributed.

Keywords: Calibration, Cross-validation, generalized model, natural forests, forest inventory.

Citation: Xelhuantzi-Carmona, J., Corral-Rivas, S., Barrera-Ramírez, R., Muñoz-Flores, H. J., & Rubio-Camacho, E. A. (2024). A mixed-effects height-diameter model for *Pinus douglasiana* Martínez in temperate forests of Jalisco, Mexico. *Agro Productividad*. https://doi.org/10.32854/agrop.v17i9.3034

Academic Editor: Jorge Cadena Iñiguez
Guest Editor: Juan Francisco Aguirre Medina

Received: May 22, 2024.
Accepted: August 03, 2024.
Published on-line: September 20, 2024.

Agro Productividad, 17(9) supplement. September. 2024. pp: 137-147.

INTRODUCTION

To foster sustainable forest management, it is crucial to employ tools that offer precise insights into the growth and development of forests and stands. For instance, incorporating both total tree-height (h) and diameter (d) into models enables accurate prediction of individual tree and stand-level volume. This approach also serves as an indicator for evaluating site productivity through estimation of dominant tree-height (López *et al.*, 2003). However, measuring total tree-height poses practical challenges, leading to increased estimation errors, time consumption, and costs during inventories. Nonetheless, employing tools such as allometric functions that model the height-diameter relationship (h-d) offers a method to estimate tree-height indirectly, thereby optimizing sampling efficiency and reducing inventory costs (Calama & Montero, 2004; Mehtätalo *et al.*, 2015).

Various functions, encompassing both linear and nonlinear forms, have been employed to develop height-diameter models. A comprehensive compilation can be found in Huang *et al.* (1992), Guzmán *et al.* (2019), and Ogana (2020). Functions that solely account for diameter are referred to as local models; however, these may not universally apply across different forest growth conditions or silvicultural treatments, as total tree-height can vary with site quality or stand density (Prodan *et al.*, 1997; Corral *et al.*, 2019; Guerra-De la Cruz *et al.*, 2019). Hence, an alternative approach involves generalized models that incorporate stand-level variables as predictors (Corral *et al.*, 2019; Rubio-Camacho *et al.*, 2022).

Generalized equations for height-diameter relationships are often fitted using the mixed-effects modeling (MEM) approach. Compared to ordinary least squares (OLS), MEMs offer greater flexibility and accuracy considering the hierarchical structure typical of forest inventories (López *et al.*, 2012). Unlike OLS models, MEMs overcome the independence assumption among observations, resulting in improved tree-height estimation (Castedo *et al.*, 2006). Moreover, MEMs can be calibrated using height measurements from a small subsample of trees. This calibration allows for the estimation of random parameters specific to a particular plot or stand, differing from those used in the model fitting process, thus enhancing their practical utility in forest management (Corral *et al.*, 2019; Rubio-Camacho *et al.*, 2022).

In the Mexican forestry sector, mixed-effects models have been sparingly utilized, primarily developed for northern (Vargas *et al.*, 2009; Corral *et al.*, 2019) and central regions (Rubio-Camacho *et al.*, 2022). However, in western Mexico, most studies do not include random effects. Therefore, there is a need to develop such tools for economically and ecologically important species in this region.

The purpose of this study was to develop a generalized model for tree-height prediction of *Pinus douglasiana* Martinez in the temperate forests of Jalisco, Mexico. The specific objectives were: i) to evaluate the effectiveness of commonly used local equations in forestry, ii) to identify and select stand variables as predictors, iii) to fit a generalized mixed-effects model, and iv) to calibrate and validate the developed mixed-effects model. The proposed model aims to provide an efficient tool applicable in western Mexico and within the natural distribution areas of the species.

MATERIALS AND METHODS

Study area

This research was conducted in the Sierra del Sur and Sierra Occidente regions of Jalisco, focusing on areas where *Pinus douglasiana* is naturally distributed. The species exhibits a broad altitudinal range from 1,100 to 2,500 meters above sea level, with optimal growth observed between 1,700 and 2,400 meters above sea level (PRODEFO, 2011). It thrives in temperatures ranging from 17 to 23 °C and typically experiences rainfall between 700 and 1600 mm annually, peaking in August and September. *Pinus douglasiana* prefers rich, well-drained soils of moderate depth (PRODEFO, 2011), and is commonly found in association with genera such as *Pinus*, *Quercus*, *Ostrya*, *Carpinus*, *Juglans*, and *Abies*, which are among the most significant (Sandoval, 2010).

Database

A total of 65 sampling plots of 50×50 m were selected where *P. douglasiana* was present (Figure 1). For each tree with a normal diameter equal to or larger than 7.5 cm, information was collected on species, tree-diameter measured at 1.3 m (d, cm) with a Jackson MS diameter tape, total tree-height (h, m) with a Suunto clinometer, among others. The database was generated by the Comisión Nacional Forestal (CONAFOR) under the project "Monitoreo Nacional Forestal: Red de Sitios Permanentes de Investigación Forestal y de Suelos" (CONAFOR, 2024). Using the collected data from each plot, the following stand variables were estimated: number of trees per hectare (N, ha^{-1}), basal area per hectare (G, ha^{-1}), quadratic mean diameter (dg, cm), dominant tree-height (H_D, cm) estimated from the 100 thickest trees per hectare. A total of 2,921 trees were measured, and a detailed description of the database is provided in Table 1 and Figure 2a.

Base model selection

The height-diameter (h-d) relationship was characterized using four commonly used local equations, which utilize diameter as the independent variable. These equations, widely applied in both national (Corral *et al.*, 2019; Rubio-Camacho *et al.*, 2022) and international (Mehtätalo *et al.*, 2015) studies, were evaluated and compared (Table 2). Each equation was fitted to the dataset using nonlinear least squares (NLS) regression.

The goodness of fit was assessed using metrics such as root mean square error (RMSE, 5), Akaike information criterion (AIC, 7), and Bayesian information criterion (BIC, 8).

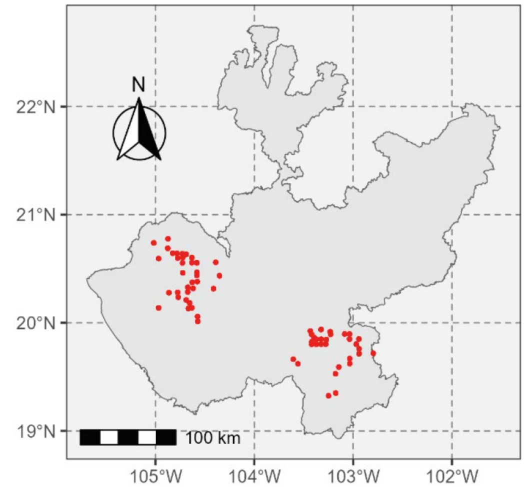

Figure 1. Location of the study area in the state of Jalisco, with sampling sites indicated in red.

Table 1. Descriptive statistics of the database.

Variables	Mean	Min.	Max.	Std.
d, cm	19.36	12.31	48.76	6.82
h, m	12.94	8.19	26.34	4.09
dg, cm	21.37	12.89	51.33	7.58
H_D	17.68	10.63	29.51	5.62
G, ha^{-1}	25.64	5.27	46.56	8.60
N, ha^{-1}	952.28	64.00	2276.00	638.48

Where: d=tree-diameter; h=total tree-height; dg=quadratic mean diameter; H_D=dominant tree-height; G=basal area per hectare; N=number of trees per hectare.

Table 2. List of equations tested for selecting the base model.

Model name	Expression	Eq.
Curtis (1967)	$h=1.3+\frac{\beta_0 \cdot d}{(1+d)^{\beta_1}}$	1
Schumacher (1939)	$h=1.3+\beta_0 \cdot \exp\left(-\beta_1 \cdot d^{-1}\right)$	2
Näslund (1936)	$h=1.3+\frac{d^2}{\left(\beta_0+\beta_1 \cdot d\right)^2}$	3
Gompertz (1825)	$h=1.3+\beta_0 \cdot \exp\left(-\beta_1 \cdot \exp\left(-\beta_2 \cdot d\right)\right)$	4

Where: h=total tree-height (m); d=tree-diameter (cm); β_i=model parameters to be estimated.

Additionally, the adjusted coefficient of determination R^2 (6) was used to measure the proportion of variance explained by each equation. The model fitting was performed using the 'nls' function in R v 4.3.2 (R Core Team, 2023).

The expressions of the goodness-of-fit statistics are as follows:

$$RMSE=\sqrt{\frac{\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2}{n-p}} \quad (5)$$

$$R^2=1-\frac{(n-1)\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2}{(n-p)\sum_{i=1}^{n}\left(y_i-\bar{y}_i\right)^2} \quad (6)$$

$$AIC=n\ln\left(\frac{SSR}{n}\right)+2p \quad (7)$$

$$BIC = n \ln\left(\frac{SSR}{n}\right) + p \ln(n) \quad (8)$$

Where: $y_i, \hat{y}_i, \bar{y}_i$ =observed, predicted and mean values, respectively, n=total number of observations, p=number of model parameters, SSR=sum of the squares of the residuals, ln=natural logarithm.

Generalized mixed-effects model

Once the most accurate local model was chosen, it was expanded by incorporating stand variables as predictors. Various combinations were tested to significantly improve the precision of total height estimation. Similar studies recommend selecting stand variables highly correlated with the parameters of the chosen local equation (Bronisz & Mehtätalo, 2020; Rubio-Camacho *et al.*, 2022; Teshome *et al.*, 2024). Finally, the resultant generalized equation was utilized to develop a mixed-effects model following the methodology proposed by Pinheiro & Bates (2013).

It is important to note that both tree-height and tree-diameter are variables typically measured within sampling plots, which are located in stands across different geographic zones. This nested structure (trees nested within plots) results in a lack of independence among observations, as data from the same sampling unit (plot) tend to be more similar (Fox *et al.*, 2001). Various methodologies have been developed to address this issue, with mixed-effects modeling approximation being one of the most widely adopted (Gregoire *et al.*, 1987; Calama and Montero, 2004). This approach simultaneously estimates fixed parameters, which are consistent across the population, and random effects specific to each plot, ensuring robust parameter estimation and associated standard errors. For a comprehensive understanding of mixed-effects models in this context, refer to Corral-Rivas *et al.* (2019), Mehtätalo *et al.* (2015), and Rubio-Camacho *et al.* (2022).

In this study, the nonlinear generalized model was linearized using the first-order Taylor series expansion and fitted using the restricted maximum likelihood procedure. Model fitting was carried out using the 'nlme' package in the R statistical software (R Core Team, 2023), employing the estimated best linear unbiased predictor (EBLUP), a method developed by Pinheiro and Bates (2000).

Mixed-effects model calibration and validation

Calibration involves estimating site-specific random effects using data independent of the model fitted. Following the approach recommended by Yang and Huang (2013), this study employed the first-order conditional expectation (FOCE) approximation for estimating random parameters. This method aligns with the iterative process used in parameter fitting to obtain the value of $\hat{b}$, as described by Lindstrom and Bates (1990).

$$\hat{b}_i = \hat{D}Z_i^T\left(\hat{R}_i + Z_i\hat{D}Z_i^T\right)^{-1}\left[y_i - f\left(x_i, \hat{\beta}, \hat{b}\right) + Z_i\hat{b}\right]$$

Where: $\hat{b}_i$ =random effect, f =non-linear function, in this case the tree h-d generalized model, $\hat{\beta}$ =fixed-effects parameter vector, i=sampling plot, $\hat{D}$ =variance-covariance matrix $(q*q)$ linked to the random effect, $\hat{R}_i$ =error term variance-covarience matrix $(m*m)$, Z_i=a matrix $m*q$ of the partial derivatives of the random effects evaluated at $\hat{b}=0$.

To calibrate the random effects for a new plot, it is necessary to sample tree heights from that plot. Various sampling alternatives have been proposed in previous studies (Corral *et al.*, 2019; Ogana *et al.*, 2020). In this research, a sample comprising the three tallest trees was selected, in line with recommendations from similar studies (Rubio-Camacho *et al.*, 2022; Teshome *et al.*, 2024). The calibration process was integrated with cross-validation techniques to estimate tree-level heights in plots not utilized in the parameter fitting process (Hastie *et al.*, 2009). Specifically, one 'plot' was omitted in each iteration to validate the fixed and calibrate the random effects simultaneously, following the approach outlined by Rubio-Camacho *et al.* (2022). This process was tested using the goodness-of-fit statistics (RMSE and R^2) to evaluate the final mixed-effects generalized model.

RESULTS AND DISCUSSION

Local models

In the first step, all four local functions were fitted using ordinary nonlinear least squares (NLS), revealing that each parameter was statistically significant at the 5% level. These functions exhibited robust fitting capabilities, explaining approximately 75% of the total height variability, with root mean square error (RMSE) values ranging from 3.1 to 3.2 meters. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) showed minimal differences across the functions (Table 3). Given the parity in goodness-of-fit statistics, further assessment involved graphical analysis of bias, AIC, and RMSE to evaluate predictive performance across diameter classes. Notably, the Gompertz function (Figure 2b) demonstrated superior performance with lower associated errors and consistent bias behavior across different diameter classes. Based on these outcomes, the Gompertz (1825) function emerges as the most appropriate model for characterizing the total height of Pinus douglasiana in Jalisco.

Generalized mixed-effects model

During the addition and combination of stand variables into the local function of Gompertz (1825), challenges with fit and convergence were encountered. The inclusion of dg, N, and G resulted in significant improvements in precision. Dominant tree-height

Table 3. Values of goodness-of-fit statistics for the local equations fitted to the height-diameter data of *P. douglasiana*.

Equation	RMSE	R^2	AIC	BIC
1	3.2109	0.74	15196.4	15214.3
2	3.2255	0.74	15223.2	15241.1
3	3.1445	0.75	15073.7	15091.6
4	3.1380	0.75	15062.4	15086.4

Where: RMSE=Root mean squared error; R^2=coefficient of determination; AIC=Akaike's information criterion; BIC=Bayesian information criterion.

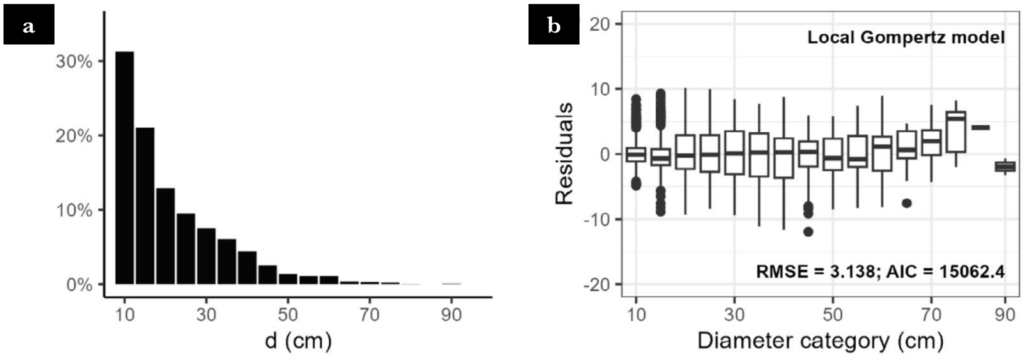

Figure 2. illustrates: a) the diameter distribution of the *Pinus douglasiana* database, and b) the residuals categorized by diameter for the local model using the Gompertz function.

(H) was deliberately omitted due to its potential for increased field sampling efforts, aligning with the practices of Bronisz & Mehtätalo (2020) and Rubio-Camacho *et al.* (2022). Ultimately, the generalized function explained approximately 80% of the variance in height with a relatively low root mean square error (RMSE), as summarized in Table 4. This finding is consistent with the results reported by Ogana (2019) for forests in Nigeria. The final expression of the generalized height-diameter function, fitted using ordinary nonlinear least squares (ONLS), is as follows:

$$h=1.3+\left(\beta_0+\beta_3\cdot\log\left(d_g\right)\right)\exp\left(-\left(\beta_1+\beta_4\cdot\left(\frac{G}{N}\right)\right)\cdot\exp\left(-\beta_2\cdot d\right)\right) \quad (9)$$

Where: h=tree height; d=tree diameter; $\beta_0 \ldots \beta_4$=fixed effects, d_g=quadratic mean diameter; G=basal area per hectare; N=number of trees per hectare.

To formulate the mixed-effects model, we initially fitted the generalized equation using all possible combinations of random effects. However, some combinations failed to converge. The combination incorporating random parameters linked to β_0 and β_1 yielded the lowest AIC and BIC values, aligning with the recommendation by Castedo-Dorado *et al.* (2006) for selecting the mixed-effects model based on these criteria. This outcome indicates that the parameters governing the asymptote and shape of the height-diameter curve vary among sites, and this variability is effectively accounted for by the model in conjunction with stand variables. The expression of the mixed-effects model is detailed in equation 10, and corresponding goodness-of-fit statistics and estimators are presented in Table 4.

$$h=1.3+\left(\beta_0+u_{0i}+\beta_3\cdot\log\left(d_g\right)\right)\exp\left(-\left(\beta_1+u_{1i}+\beta_4\cdot\left(\frac{G}{N}\right)\right)\cdot\exp\left(-\beta_2\cdot d\right)\right) \quad (10)$$

Where: h=tree height; d=tree diameter; $\beta_0 \ldots \beta_4$=fixed effects; u_{0i} y u_{1i}=random effects on β_0 y β_1; d_g=quadratic mean diameter; G=basal area per hectare; N=number of trees per hectare.

Table 4. Parameter values and goodness-of-fit statistics of the generalized equations for *Pinus douglasiana*.

Components	Base model (NLS)	Mixed model
Fixed effects		
β_0	−5.70 (1.950)	−2.76 (6.053)
β_1	1.84 (0.075)	1.57 (0.164)
β_2	0.08 (0.003)	0.07 (0.002)
β_3	9.01 (0.554)	7.7 (1.850)
β_4	−8.67 (2.175)	−9.89 (2.872)
Random effects variance		
sd(u_{0i})	-	4.11
sd(u_{1i})	-	0.52
Cor(u_{0i}, u_{1i})	-	0.37
Model performance		
RMSE	2.98	2.09
R^2	0.78	0.89
AIC	14767.06	12678.45
BIC	14802.97	12714.37
Cross-Calibration-Validation (mixed-effects model)		
Sub-sample trees (n)	-	3
RMSE	-	2.59
R^2	-	0.83

Where: β_0... β_4=fixed parameters; sd(u_{0i}) y sd(u_{1i})=Standard deviation of the random effects; Cor=Random effects correlation; RMSE=Root mean squared error; R^2=coefficient of determination; AIC=Akaike's information criterion; BIC=Bayesian information criterion.

The development of the mixed model demonstrates an 11.25% increase in explained variability compared to the base generalized model, confirming significant variability among plots (Table 4). Examination of residual plots for the mixed model (Figure 3) indicates consistent variance homogeneity across the entire range of predicted values.

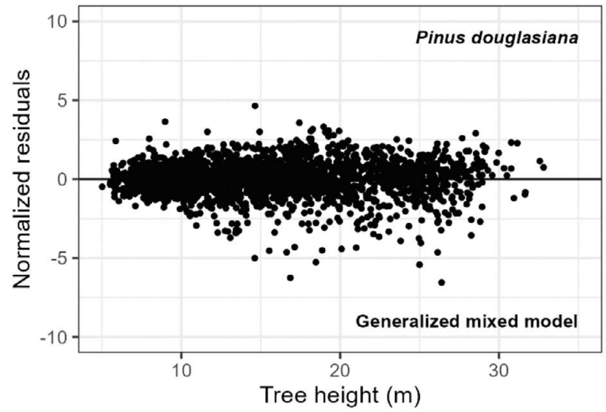

Figure 3. Distribution of residuals from the final model.

This, coupled with our use of cross-sectional data where observations within the same plot are uncorrelated, suggests that inclusion of a "weight factor" in the error variance of the variance-covariance matrix for each study plot is unnecessary (Fang & Bailey, 2001; Corral *et al.*, 2019).

Model calibration and validation

In this study, calibration and validation of the mixed-effects model were conducted simultaneously. To optimize sampling efforts and reduce associated inventory costs, the height of the three thickest trees in each plot or sampling site was measured (Table 4). To assess the improvement in goodness-of-fit statistics with the selected calibration approach, the accuracy achieved relative to the fit of the generalized model by ordinary nonlinear least squares (NLS) was evaluated. The RMSE value showed marginal reduction, and absence of collinearity in residuals supports the recommendation of using the generalized mixed model with random parameter estimation based on a subsample of three trees.

Several studies have compared different tree combinations for calibration (Bronisz & Mehtätalo, 2020; Ogana *et al.*, 2020; Teshome *et al.*, 2024). However, in line with the recommendation by Rubio-Camacho *et al.* (2022), the generalized mixed model offers significant practical advantages for field applications, requiring measurement of only three subsample trees.

CONCLUSIONS

The Gompertz model emerged as the most effective predictor of the tree-height for *P. douglasiana* trees among the local models assessed. Incorporating stand variables (N, G, and dg) notably enhanced height predictions. Furthermore, adding the random effects successfully captured plot-to-plot variation, resulting in improved height estimations.

Regarding calibration, measuring the heights of the three thickest trees proved to be a robust sampling method for estimating random effects, thereby enhancing height predictions for trees in new plots or stands. However, in cases where this subsample is unavailable, the mixed-effects model remains valuable by setting random effects to zero. However, for optimal accuracy, the calibration technique is recommended when applied to new plots or sites. The model developed in this study is expected to assist foresters in reducing inventory costs and accurately estimating missing or erroneous height data within inventory databases. Ultimately, this model provides a reliable tool for height estimation in natural forests of *P. douglasiana* in Jalisco state and other regions where the species is naturally distributed.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Comisión Nacional Forestal and the Universidad Juárez del Estado de Durango for providing access to the database generated through the Red de Sitios Permanentes de Investigación Forestal y de Suelos - 2024, which formed the basis of this study. Financial support from INIFAP through project number 2-1.6-12521135910-F-M.2-1 is also acknowledged.

The research station 'Altos de Jalisco', now known as 'Campo Experimental Centro-Altos de Jalisco', was established in 1974. We acknowledge the institution and its dedicated personnel for their unwavering support

and valuable contributions that have benefitted the people of Mexico, marking a significant milestone over decades. This manuscript stands as our tribute, commemorating 50 years of their remarkable achievements.

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