






Income optimization of a biodiesel, glycerin and soap company using waste cooking oil

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ABSTRACT

Objective: Optimize a production unit for biodiesel, glycerin, and soap using a mathematical programming model (MPM). Determine the optimal quantities of waste cooking oil from different sources (ACD_1 =glycerin and biodiesel; ACD_2 =soap) to maximize the income of a biodiesel production company.

Design/methodology/approach: To assess the availability of ACD inputs, stratified sampling was conducted from February 8 to 19, 2021, measuring the ACD generation of restaurants in downtown Texcoco, State of Mexico. The analysis was conducted using an economic approach for a biofuel-producing company, considering 10 decision variables in the objective function (OF), subject to 18 constraints. These variables were based on the technical requirements to produce 1 liter of biodiesel and a 400 g bar of soap, using technical coefficients (input-product). Excel[®] Solver[®] was used to generate a sensitivity report and analyze shadow prices (SP) and reduced costs (RC).

Results: Monthly revenues exceeding \$140,000 MXN were obtained. ACD_2 emerged as the scarcest and most valuable input within the model.

Limitations on study/implications: Proper management of ACD disposal can generate income through the production of biodiesel, glycerin, and soap.

Findings/conclusions: MPM proves to be the most efficient method for utilizing kitchen waste and converting it into valuable products.

Keywords: Waste cooking oil, biodiesel, mathematical programming, optimization.

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INTRODUCTION

Edible vegetable or animal oils are a fundamental dietary component in the Mexican diet. After frying and cooking processes, waste cooking oil (ACD) is generated (Banerji *et al.*, 2022). At high temperatures (160-200 °C), vegetable oils composed of triacylglycerols undergo physical and chemical modifications, resulting in toxic compounds formed through oxidation, hydrolysis, and polymerization of triglycerides (Tsoutsos *et al.*, 2016). According to González and González (2023), 1 liter of used cooking oil can contaminate approximately 40,000 liters of drinking water. Furthermore, once cooled, the oil solidifies and can clog pipes and drains. Improper disposal of ACD poses serious environmental issues. The compounds produced from the degradation of cooking oils have harmful effects on

human health and are known to be mutagenic, carcinogenic, neurotoxic, and hepatotoxic, among others (Tsoutsos *et al.*, 2016). This highlights that improper ACD disposal can lead to environmental pollution and public health problems (Lopes *et al.*, 2020). There is a pressing need to implement alternatives for the reuse and final disposal of these residues in order to give new life to what is typically considered single-use waste (Tsoutsos *et al.*, 2016). ACD can be repurposed through transformation, for which the simplex method an algorithm used to optimize the value of the objective function (OF) while considering defined constraints proves useful. This is an iterative process that improves the OF in each step until reaching the optimal solution (Alvarado-Boirivant, 2011). In this research, an MPM was developed to maximize the net income of a company producing biodiesel, glycerin, and soap, making efficient use of ACD as a limited resource.

MATERIALS AND METHODS

The ACD was classified into two types based on its origin and quality, according to culinary practices. The quantity of ACD generated was estimated by production strata in the central area of the municipality of Texcoco, State of Mexico, Mexico, located at coordinates 19° 30’ 42.98” N, 98° 52’ 58.55” W.

A stratified sampling was conducted from February 8 to February 19, 2021: ACD₁ was obtained from restaurants (n=16), taco stands and Mexican snacks vendors (n=12), and local diners (n=5) to produce biodiesel and glycerin; ACD₂ was collected from fast food establishments and bars (n=13), as well as donut and churro vendors (n=5) for soap production. The sample size was based on Santoyo *et al.* (2002), totaling 51 units (22% of the total population). The model includes percentage-based constraints on the availability of ACD types, according to their participation in final production. A total of ACD₁=18,833 L/month was recorded, of which 82% came from restaurants, 15% from taco stands, and 3% from local diners. ACD₂=14,147 L/month was primarily sourced from fast food vendors, contributing 46.3%. Using a mathematical programming model (MPM), an optimal profit-maximizing solution was sought through the Simplex method via Microsoft Excel[®] and Solver[®]. The model design required a literature review to obtain technical coefficients that identified the necessary inputs for producing biodiesel, glycerin, and soap. The general form of the objective function (OF) is mathematically expressed as described by Anderson *et al.* (2013) and Alvarado-Boirivant (2011):

$$\text{Maximize } Z = \sum_{j=1}^n c_j * x_j \quad \text{where } j=1 \text{ up to } n \tag{1}$$

Subject to 18 constraints and 10 decision variables:

$$\begin{aligned} a_{11} X_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1\ 10} x_{10} &\leq \geq b_1 \\ a_{21} X_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{2\ 10} x_{10} &\leq \geq b_2 \\ a_{31} X_1 + a_{32} x_2 + a_{33} x_3 + \dots + a_{3\ 10} x_{10} &\leq \geq b_3 \\ \cdot &\cdot \\ \cdot &\cdot \\ a_{18\ 1} X_1 + a_{18\ 2} x_2 + a_{18\ 3} x_3 + \dots + a_{18\ 10} x_{10} &\leq \geq b_{18} \end{aligned} \tag{2}$$

$$\text{And non-negativity restrictions } x_j \geq 0 \tag{3}$$

Where c =net price coefficient of OF, a =technical coefficient, x =decision variable, b =available resource. Using the sensitivity report in Excel® Solver®, shadow prices (SP) and reduced costs (RC) were analyzed. Optimized production and sales of biodiesel, glycerin, and soaps were considered (Table 1).

In this way the OF is:

$$\begin{aligned} \text{MAX } Z = & ACD_{1r} * (-8) + ACD_{1t} * (-8) + ACD_{1ce} * (-8) + ACD_{2cr} * (-8) + ACD_{2p} \\ & * (-8) + BIO * (-13.48) + JB * (-11.1) + Gl * 30 + J * 50 + B * 24 \end{aligned} \tag{4}$$

The technical coefficients were obtained from Tacias *et al.* (2016). Through the transesterification of fats and oils, 1 L of ACD reacting with 0.376 mL of methanol (CH₃OH) in a basic environment and under the action of a catalyst such as 0.09 g of potassium hydroxide (KOH), produces 1 L of biodiesel and 0.361 mL of glycerin. For the production of a 400 g bar of soap, 253 mL of ACD, 139 g of sodium hydroxide (NaOH), 4 g of colorant, and 4 g of fragrance are required. The study also notes that the acid index (AI) and the content of free fatty acids in the oil are key parameters that determine the feasibility of ACD for biodiesel production. The AI was measured based on the Mexican standard for determining free fatty acids (NMX-F-101-SCFI2012, S.E. 2012), calculating the free fatty acids as oleic acid (%) using the following formula:

$$\text{Free fatty acids such as oleic, in \%} = \frac{V \times N \times 28.2}{pm} \tag{5}$$

Where: V =milliliters of sodium hydroxide solution used to titrate the sample. N =normality of the sodium hydroxide solution. pm =mass of the sample in grams.

Table 1. OF decision variables.

Variable	Abbreviation	Notation	Coefficient	Element in OF
Waste Cooking Oil from Restaurants	ACD _{1r}	X ₁	c ₁	c ₁ *X ₁
Waste Cooking Oil from Mexican taquerias and snack bars	ACD _{1t}	X ₂	c ₂	c ₂ *X ₂
Waste Cooking Oil from Commercial Kitchens	ACD _{1ce}	X ₃	c ₃	c ₃ *X ₃
Waste Cooking Oil from Fast Food and Snack Bars	ACD _{2cr}	X ₄	c ₄	c ₄ *X ₄
Waste Cooking Oil from Potatoes and Donuts	ACD _{2p}	X ₅	c ₅	c ₅ *X ₅
Biodiesel Production	BIO	X ₆	c ₆	c ₆ *X ₆
Bar Soap Production	JB	X ₇	c ₇	c ₇ *X ₇
Glycerin Sales	Gl	X ₈	c ₈	c ₈ *X ₈
Kg of Soap Sales	J	X ₉	c ₉	c ₉ *X ₉
Biodiesel Sales from All Levels	B	X ₁₀	c ₁₀	c ₁₀ *X ₁₀

Source: Prepared by the authors. c_i =net price=product price_{*i*}* quantity produced_{*i*}-variable cost_{*i*}

To measure the acid index (AI), the following materials were used: a) accurately standardized sodium hydroxide solution; b) 95% (v/v) ethyl alcohol, neutralized at the time of use with 0.1 N sodium hydroxide using phenolphthalein as an indicator; c) 1.0% alcoholic phenolphthalein indicator solution; d) 500 mL Erlenmeyer flask; e) 20 cm³ volumetric pipette; f) 50 cm³ burette graduated in 0.1 cm³; g) electromagnetic stirrer; h) beaker; i) graduated cylinder; and j) 30-gauge stainless steel mesh. ACD₁ with an AI < 2 was used for biodiesel and glycerin production, while ACD₂ with an AI > 2 was used for soap production (Table 2).

The MPM was constrained with the following algorithms:

$$\text{Liters of ACD}_1: (1X_1 + 1X_2 + 1X_3) \leq 18833$$

$$\text{Liters of ACD}_2: (1X_4 + 1X_5) \leq 14047$$

$$\text{Liters of methanol: } (0.376X_6) \leq 12362.88$$

$$\text{Potassium hydroxide catalyst (KOH) kg: } (0.09X_6) \leq 295.2$$

$$\text{Sodium hydroxide catalyst (NaOH) kg: } (0.139X_7) \leq 18281$$

$$\text{Soap coloring g: } (0.04X_7) \leq 5260.8$$

$$\text{Fragrance soaps g: } (0.04X_7) \leq 5260.8$$

$$\text{Labor hours: } (1X_6 + 1X_7) \leq 10000$$

$$\text{Energy Kilowatts: } (0.0185X_6 + 0.0041X_7) \leq 32880.368$$

$$\text{Distilled water liters: } (0.05X_6) \leq 274$$

$$\text{ACD}_1 \text{ biodiesel restaurant L: } (0.188X_1 - 0.812X_2 - 0.1821X_3) \leq 0$$

$$\text{ACD}_1 \text{ taco shop for biodiesel L: } (-0.1581X_1 - 0.8419X_2 - 0.1581X_3) \leq 0$$

$$\text{ACD}_2 \text{ fast food and bars L: } (0.5375X_4 - 0.4625X_5) \leq 0$$

$$\text{Biodiesel transfer L: } (-1X_1 - 1X_2 - 1X_3 - 1X_6) \leq 0$$

$$\text{Soap transfer L: } (-1X_4 - 1X_5 + 2.5X_7) \leq 0$$

$$\text{Biodiesel for sale L: } (-1X_6 - 1X_{10}) \leq 0$$

$$\text{Glycerin for sale L: } (-0.3661X_6 - 1X_8) \leq 0$$

$$\text{Wind soap L: } (-1X_7 + 1X_9) \leq 0$$

Table 2. Average annual acidity index by stratum.

Stratum	Density Kg/m ³	Goo mm ² /s	Free Fatty Acids %	Acid number mg KOH/g
Formal Restaurants	912	41.5	0.9	1.78
Taquerias and Mexican Snacks	860	46.1	1.57	1.16
Cheap Kitchens	909	34.9	0.99	1.98
Fast Food and Bars	890	44.6	1.72	5.42
Potatoes and Churros	918	56.9	0.58	3.13
Average	897.8	44.81	1.35	2.7

Source: Prepared by the authors.

RESULTS AND DISCUSSION

From the MPM, the total Net Income obtained was \$149,983.32 MXN per month from the activities of the biodiesel, glycerin, and soap production company. All variables were selected by the model; therefore, each had a Reduced Cost (RC) of 0 (Table 3).

When the objective is maximization, the opportunity cost or shadow price (SP) represents the benefit forgone by not having an additional unit of a resource, and thus this result can be interpreted as a market price (Anderson *et al.*, 2013). The catalyst KOH has an SP of \$148.33 MXN/L, as this input is essential for the production of biodiesel and glycerin. The SPs corresponding to transfers match the market price in the MPM, while the remaining inputs have an SP=0, indicating they are still available (Table 4).

Bierkens *et al.* (2019) mention that market prices rarely constitute a valid indicator of the social value of resources, due to the various reasons that cause such markets to not operate under perfect competition. One alternative is the so-called shadow prices (SP), which indicate the marginal cost of production. The model yielded an SP of \$7.56 MXN/L for ACD₂, a figure close to its market price, which according to the surveys conducted was \$8.00 MXN; for ACD₁, its SP=0. According to Ortiz and Caicedo (2015), constraints with an SP=0 indicate that having a greater quantity of these resources will not result in an increase in total net income. Therefore, the model indicates that ACD₂ is more valuable than ACD₁, and it is more feasible for the company to focus on soap production rather than biodiesel production.

Table 3. Optimal value of activities that maximize the objective function.

Variable	Optimized quantity	RC
ACD _{1r}	2663.36	0
ACD _{2t}	518.57	0
ACD _{1ce}	98.07	0
ACD _{2cr}	6496.74	0
ACD _{2p}	7550.26	0
BIO	3280	0
JB	5618.8	0
Gl	1184.08	0
Jk	5618.8	0
B	3280	0

Source: Data obtained directly from the MPM.

CONCLUSIONS

The model maximized its profits and a monthly profitability exceeding \$140,000 MXN was determined. This income increases based on the addition of scarce resources, such as ACD₂ and KOH. Therefore, it is concluded that ACD₂ is more important for this model, as its availability allows for soap production. The obtained shadow prices (SP) reflect the scarcity of resources the higher the value, the scarcer the resource is considered, and thus it holds a higher value as an alternative for resource valuation.

Table 4. Shadow prices of the MPM.

Restriction	Resource (inputs)	SP
1	ACD ₁	0
2	ACD ₂	7.56
3	Biodiesel Transfer	8
4	Soap Transfer	15.56
5	Methanol	0
6	Catalyst (KOH)	148.33
7	Catalyst (NaOH)	0
8	Dye	0
9	Fragrance	0
10	Labor	0
11	Electricity	0
12	Distilled Water	0
13	Transfer of Produced Biodiesel for Sale	24
14	Transfer of Glycerin for Sale	30
15	Transfer of Soap for Sale	50
16	Restriction on Restaurant ACD	0
17	Restriction on Taco Shops	0
18	ACD food bar restriction	0

Source: Data obtained directly from the MPM.

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