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Dynamic stochastic model of allometric equations and cumulative distribution for biomass-carbon in *Pinus hartwegii* Lindl., facing climate change

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ABSTRACT

Objective: to construct a dynamic stochastic model with validated biospheric-interaction, estimating allometric equations for the total volumetric increase and cumulative distribution of biomass for *Pinus hartwegii* Lindl in the states of Mexico and Puebla, considering climate change.

Design/Methodology/Approach: the methodology included the use of SiBiFor numerical databases, NASA Power data, Ordinary Least Squares mathematical models, the Random-Forest software, Ridge model with regression, R algorithms and Newton volumetric estimation equations.

Results: estimated allometric equations were obtained for the total volume of trees in 2023, highlighting the importance of linear regression models and the validity of the variables used. Newton's mathematical equations and theoretical models for excurrent tree-form types were found to have the best accuracy in estimating the total volume of the barked tree.

Limitations of the study/Implications: this study has limitations in terms of generalization to other forest types and the availability of data in Mexico. However, it highlights the importance of understanding forest responses to environmental changes and the need for validated dynamic stochastic models to estimate allometric equations and assess carbon sequestration.

Findings/Conclusions: this study highlights the importance of understanding and assessing the carbon storage capacity of forests, especially in the context of climate change. In addition, it underlines the usefulness of linear regression models and variable validation to estimate carbon sequestration in *Pinus hartwegii* forests.

Keywords: stochastic modeling, biomass-carbon, *Pinus hartwegii*, climate change, Random Forest.

INTRODUCTION

Developing the theoretical-practical mathematical modeling capacity based on the dialectical relationship between the logics of mathematics and engineering responds to the

interest of solving situations specific to the engineering profession (Torrecilla Díaz *et al.*, 2015). A concrete example is a stochastic mathematical model, defined as a mathematical abstraction of an empirical process governed by probabilistic laws and set as a quantitative model in which more than one state of nature exists. This model is created to calculate the conditional outcomes of decision alternatives in each state (Vega, 2022). An example of the practical application of these models is the reduction of natural vegetation and the importance of forest masses as CO_2 reservoirs based on their sensitivity to climate change at a global scale (Sandoval, 2020).

Each species or plant population requires specific conditions of temperature, humidity and light for its physiological processes. If it exceeds its degree of tolerance this will be reflected in the alteration of species distribution, which is more severe in individuals of mountainous ecosystems. Even though there are extensive investigations with criteria of climate change, wood traits and molecular variations, quantification of organic carbon stored in soil per ha^{-1}, physical-chemical properties of wood, dendro-epidometric measurements, development, validation, simulation with mathematical modeling, they are included in few forestry investigations. Due to this; the theory of stochastic processes includes random vectors of infinite dimension or arbitrary infinite collections of arbitrary variables for dynamic model analysis, which account for the evolution of those variables over time (Valderrama, 2009), yet, they are scarce in Mexico.

The cumulative biomass is conditioned simultaneously by the biocoenotic and abiocoenotic components in situ at any instant; thus describing temporal evolutionary displacement of *P. hartwegii* Lindl forests in those Mexican highland states. In the face of climate change, mitigation is of great relevance. Static and dynamic biometric models, deterministic as well as stochastic, are essential to understand the dynamics and to evaluate the carbon storage capacity of forests (Santiago *et al.*, 2020), which underlines the need to contribute to this knowledge (Martínez-Luna *et al.*, 2020; Ortega Rivera, 2021).

Based on this, the objective of this research was to construct a dynamic stochastic model with a validated biosphere-interaction modulus that would estimate allometric equations of the total volumetric increase, and the cumulative distribution of biomass for *Pinus hartwegii* Lindl, to evaluate carbon sequestration in the states of Mexico and Puebla.

MATERIALS AND METHODS

The study was implemented in *Pinus hartwegii* Lindl forests in the states of Mexico and Puebla. The numerical databases of the SiBiFor Project were used (Vargas-Larreta *et al.*, 2017Guerrero, Jalisco, Oaxaca, Michoacán, Puebla, State of Mexico, Hidalgo, Tlaxcala, Veracruz and Quintana Roo. Field data collection was done by using destructive and non-destructive sampling in each Regional Forest Management Unit (RFMU) considering the 2013-2015 time series and the NASA methodology based on the POWER Project (Al-Kilani *et al.* 2021; NASA, 2021; POWER 2021) to analyze significant climatic variables that influence the growth of *Pinus hartwegii* forests. Measurements of normal diameter (DN), total height, and commercial height of the trees were analyzed.

The numerical databases in each time (2013-2015) were taken in the field by the technical staff of SiBiFor for the timber species *Pinus hartwegii* Lindl. in the states of

Mexico and Puebla. Then, they were divided into the categories "commercial volumetric dimension", with dasometric measurements of dominant and codominant trees and "non-commercial volumetric dimension", with dasometric measurements of suppressed trees, with the aim of estimating a model for each category separately, since it would not justify its use only with the statistical criterion of R^2 and, consequently, the predictions would be biased. Based on this, with the use of Newton's mathematical equation to estimate timber volumes; theoretical models for excurrent dendrometric types (volume per timber unit, *i.e.* logs); allometric equations used in the field by the governmental institutions of the Mexican states; the Ordinary Least Squares method (OLS) and algorithms in R; the allometric equations for "commercial volumetric dimension" of *Pinus hartwegii* Lindl were estimated using mathematical models in the states of Mexico and Puebla.

A dynamic stochastic model of allometric equations and cumulative distribution was used to estimate carbon sequestration in *Pinus hartwegii* forests. Different statistical methods and algorithms were applied to validate the model, including regression analysis, and the Ordinary Least Squares method (OLS) (Briggs & MacCallum, 2003; Pretzsch, 2009).

In this research, five exploratory techniques of the Graphical Method were used; the scatter plots of the squared residuals of the model against the values of the variables, both endogenous and exogenous were analized, searching for systematic patterns that could reveal heteroskedasticity. In case the residues are homoscedastic, the scatter plot would take the form of an almost constant horizontal band. If patterns are observed in the graphs, there would be evidence of dependence between the squared residuals and the variables, moreover, it would be known which variable is the cause of this dependence. The graphic methods used were residual graph, residuals *vs.* fitted graph, Q-Q Normal Chart, Scale Location Model Chart, and Residuals *vs.* Laverage Chart.

Since the Ridge regression is a biased estimate that starts from the solution of the Least Squares (LS) regression, it is vital to establish the conditions for which the central Student's t-distribution used in the hypothesis test in CM, is also applicable to the regression. The proof of this important result is presented in this research. The method allows detecting multicollinearity within a regression model of the type $Y = X^t \hat{\beta} + \epsilon$, that is used to operate such biased models.

Random Forest is an assembly-based machine learning algorithm that combines multiple decision-trees to improve accuracy and to reduce variance in predictions (Breiman, 2001) cited in Alaminos (2023). Along recent years, Random Forest has been used in many practical applications and has proven effective in a wide variety of problems.

Data for DN, total height, and commercial tree height were used to develop allometric equations and cumulative distributions to estimate carbon sequestration in *Pinus hartwegii* forests. In addition, calculations of timber volume were applied by OLS criterion and Ridge regression (Hoerl & Kennard ,1970; Piña-Monarrez *et al.*, 2007) to assess the carbon storage capacity of forests.

Statistical tests were performed and graphical methods were used to validate the proposed model. Statistical software R was used through the Random Forest procedure (Mohamed *et al.*, 2024) to run data analysis and output generation.

RESULTS AND DISCUSSION

The use of climate data obtained through NASA POWER project (NASA, 2021; POWER 2021), including solar radiation, temperature and humidity, were significant variables for the growth of *Pinus hartwegii* forests. These data were fundamental for the estimation of allometric equations of total volumetric increase and cumulative distribution of biomass, as well as for assessing carbon sequestration in the forests studied.

Table 1 presents the allometric equations estimated by OLS (R software) and SiBiFor methodology (Vargas-Larreta *et al.*, 2017) for the *Pinus hartwegii* forests in the states of Mexico and Puebla.

Table 2 includes the allometric equations estimated by OLS (Ridge Model) for *Pinus hartwegii* considering those climatic variables that are relevant. These equations were developed using climate data provided by the POWER project. It was found that the linear allometric equations estimated by OLS with the Ridge Model provide a better volumetric approximation to climate change.

Table 3 shows the predictions of the numerical values representing the direct relationship between the apparent and specific climate conditions and their effects on *Pinus hartwegii* Lindl during the time span between 2022 and 2024 in the territories of the states of Mexico and Puebla.

According to the graphic validations, some models met the evaluation criteria. The estimated allometric equations "non-commercial volumetric dimension" of *Pinus hartwegii* Lindl using mathematical models in the states of Mexico and Puebla showed low statistics under analysis of variance criteria. This occurred because dasometric measurements of suppressed trees differ markedly. According to the validation charts, not all of the models met the evaluation criteria; but, based on field cross-validation, the traditional mathematical equation with the best accuracy in estimating the total volume of barked trees for *Pinus hartwegii* Lindl in the states of Mexico and Puebla is Newton with an evaluation of 95% and 98%, respectively. Since it uses the largest, middle and smallest diameters of logs while the allometric equation estimated by mathematical models showed the best estimate of the total tree volume with bark for this timber forest species. The allometric equation estimated by mathematical models with better estimation of the TAV cc was the theoretical model for excurrent tree form types (volume per logs) in the states of Mexico and Puebla with an evaluation of 90.5% and 95%, respectively.

Based on the use of the SiBiFor numerical databases from a given time (2013-2015) for the timber species *Pinus hartwegii* Lindl., the application of the 'Random Forest' for R package to interpolate and estimate the non-existent information of the variable "Edad_no_anillos" in the UMAFOR Num.1508 in the state of Mexico and Num. 2105 in the state of Puebla; the use of Power data, in monthly time series format of (1980-2021) from NASA methodology; and the discernment of which climatic variables are relevant in the volumetric growth of a tree, allometric equations of "commercial volumetric dimension" with significant climatic variables for *Pinus hartwegii* Lindl. were estimated in the states of Mexico and Puebla.

According to Hoerl & Kennard (1970), this occurs because the advantage of Ridge regression over least squares has its origin in the bias-variance compensation. As λ increases,

Table 1. Summary of allometric equations estimated by Ordinary Least Squares (OLS) with R and SiBiFor for *Pinus hartwegii* for the state of Mexico and Puebla.

State	Volumetric	Method of fitting							
		OLS					SiBiFor		
		Equation	**Multiple Square R**	**Square R Fitted**	**F- statistic**	**p-value**	**UMAFOR**	**Equation**	**R^2**
México	Newton	log(Vol)=−0.12 + 0 log_h + 0.33 log_diam	0.9827	0.9826	8037	<2.2e-16	1508	0.00005*(Diam^1.97258)*(Alt^0.92797)	0.9618
	Theoretical (logs)	log(Vol)=0.11 + 0.12 log_h + 0.25 log_diam	0.9748	0.9746	5463	<2.2e-16	1510	0.00004*(Diam^2.15224)*(Alt^0.78917)	0.9585
	State	log(Vol)=−9.94 + 0.85 log_h + 2.05 log_diam	0.9971	0.997	43730	<2.2e-16			
Puebla	Newton	log(Vol)=−0.07 + 0.05 log_h + 0.27 log_diam	0.9627	0.9624	3831	<2.2e-16	2101	0.00006*(Diam^1.99376)*(Alt^0.90054)	0.9721
	Theoretical (logs)	log(Vol)=0.12 + 0.15 log_h + 0.22 log_diam	0.9642	0.964	4000	<2.2e-16	2105	0.00004*(Diam^2.20234)*(Alt^0.66536)	0.9772
	State	log(Vol)=−8.18 + 0.61 log_h + 1.79 log_diam	0.9709	0.9707	4889	<2.2e-16	2108	EXP(-9.63495649+1.86670523*LN(diam)+0.99551381*LN(alt))	0.9939

*Method of fitting: it indicates the method used to fit the allometric equations, either OLS or SiBiFor; State/State: it shows the geographic location for which the allometric equations were estimated; Volumetric method: it describes the method used to calculate the volume of trees (Newton; Theoretical by log [as unit of timber] and the one commonly used by State); Equation: it presents the allometric equation resulting from the fit done; R^2: explained variance percentage; Multiple Square R: it is a measure of the proportion of variability in the dependent variable that is explained by the model, it indicates how well the model fits the data; Square R fitted: This is similar to multiple square R, but it takes into account the number of predictors in the model; it helps to compare models with different numbers of predictors; F-statistic: it is the value that test the global significance of the regression model; p-value: this is the value of p associated with the F-statistic, it indicates whether the model is significant.

Table 2. Summary of allometric equations estimated by MCO with Ridge Model for *Pinus hartwegii* with significant climatic variables.

Concept	Estado de México			Puebla		
	Method for Commercial volumetric dimension			Method for Commercial volumetric dimension		
	Newton	Theoretical (logs)	State	Newton	Theoretical (logs)	State
Explained Variance percentage (R^2)	0.951499	0.939338	0.928746	0.93038	0.934537	0.93575
(Intercept)	39.859663	50.64726	−64.30451	105.415	154.6166	−131.9211
Altura total (h)	0.01572	0.045025	0.031397	0.00809	0.027984	0.013225
Diámetro normal (diam)	0.025014	0.026166	0.028203	0.01493	0.018439	0.047649
Earth Skin Temperature (°C) (TS)	−0.010908	−0.012784	0.001895	−0.003	−0.005506	0.007905
Temperature at 2 meters (°C) (T2M)	−0.010975	−0.013165	0.007274	−0.0123	−0.020297	0.021197
Top−of−Atmosphere Shortwave Downward Irradiance (TOA_SW_DWN)	−1.924237	−2.505947	2.253705	−3.4611	−4.817098	3.986277
All Sky Surface PAR Total (ALLSKY_SFC_PAR_TOT)	0.002949	0.004038	−0.003183	0.00437	0.00509	−0.003492
Clear Sky Surface PAR Total (CLRSKY_SFC_PAR_TOT)	0.006476	0.008882	−0.004877	0.00597	0.005833	−0.003405
Surface Pressure (PS)	0.338027	0.454903	−0.163521	0.17873	0.146928	−0.074175
Specific Humidity at 2 meters (QV2M)	0.045728	0.059961	−0.013913	0.0136	0.009449	−0.004173
Relative Humidity at 2 meters (RH2M)	0.004126	0.005259	−0.000788	0.00069	0.000407	−0.000158
Root Zone Soil Wetness (GWETROOT)	0.229238	0.283766	−0.027523	0.02171	0.010888	−0.003766
Precipitation Corrected (PRECTOTCORR)	−0.029698	−0.035689	0.002254	−0.0016	−0.000687	0.000212
"Edad_no_anillos" variable	0.006924	0.006797	−0.000682	0.00566	0.007813	0.015973

*Concept: describing each term used by OLS with Ridge Model; State (Mexico or Puebla): it indicates the Mexican state in which the allometric equations were estimated; Method for Commercial Volumetric Dimension: it shows the method used to calculate the commercial volumetric dimension of trees (Newton; Theoretical –by logs– or by State); Explained variance percentage (R^2); Intercept: it refers to the intercept of the allometric equation, which is the value of the dependent variable when all independent variables are equal to zero; Total height (h), Normal diameter (DN) and other climatic variables: estimated coefficients for each of the independent variables included in the allometric model, indicating how each variable contributes to the calculation of tree volume.

the variance of the Ridge regression decreases, but the bias increases. In addition, in the least squares coefficient estimates, which correspond to the Ridge regression with $\lambda=0$, the variance is high but there is no bias. Also, the Paterson Productivity Index C.P.V. indicated that climate is one of the essential elements in forest production, tough Paterson limited his study to temperature, humidity, timing of the growth period and intensity of radiation, since climate is the main ecological factor on a regional scale and its influence is mainly expressed in changes in the physiognomy of vegetation and floristic composition (Walter, 1977; Petagna Del Río 1993; Gliessman *et al.*,1998).

Table 3. Estimates of the linear coefficients of significant climate variables in the period 2022-2024 for *Pinus hartwegii* Lindl in the states of Mexico and Puebla.

Climate variable	Linear coefficients estimated by Mexican state					
	México			Puebla		
	2022	2023	2024	2022	2023	2024
Earth Skin Temperature (C) (TS)	17.3631605	17.2562204	17.2075609	15.0913378	15.093867	15.09378
Temperature at 2 meters (C) (T2M)	16.0688180	15.9727000	15.9824568	13.9756634	13.9538095	13.9546881
Top-Of-Atmosphere Shortwave Downward Irradiance (TOA_SW_DWN)	34.0226381	34.0161901	34.0216112	34.2100035	34.2068739	34.2110193
All Sky Surface PAR Total (ALLSKY_SFC_PAR_TOT)	102.2113483	101.4544898	101.8056317	107.4957497	107.3481481	107.4148577
Clear Sky Surface PAR Total (CLRSKY_SFC_PAR_TOT)	122.7126980	122.6059464	122.5449116	125.1707133	125.4006844	125.3800238
Surface Pressure (PS)	77.2082785	77.2019272	77.2015360	76.6635899	76.6581183	76.6585532
Specific Humidity at 2 meters (QV2M)	8.4555264	8.5050894	8.4706968	9.2473988	9.2468102	9.2464988
Relative Humidity at 2 meters (RH2M)	61.9231341	62.2058595	62.3261776	72.2132697	72.4343616	72.074441
Root Zone Soil Wetness (GWETROOT)	0.4857650	0.4844374	0.4903292	0.4207275	0.4195527	0.4167194
Precipitation Corrected (PRECTOTCORR)	1.8141824	1.7827351	1.8955158	2.6239403	2.6892061	2.6296263

*Climate variable: name of the measured climate variable, such as temperature, solar radiation, atmospheric pressure, humidity, etc.; Linear coefficients estimated by Mexican state: numerical values represent the estimates of linear coefficients for each climate variable in the years 2022, 2023 and 2024, respectively, for the Mexican states of Mexico and Puebla.

Finally, regarding the estimation of timber volume for *Pinus hartwegii* Lindl based on linear allometric equations estimated by OLS with the Ridge Model for normal diameter, height and significant climatic variables, the simulations carried out with the linear diameter-height coefficients per tree located in the UMAFOR, and the significant climatic variables indicated a better volumetric approximation by Newton mathematical equations, and theoretical models (volume by timber unit, logs) for excurrent dendrometric types. According to Martínez-Luna *et al.* (2020) "if a coefficient of 50.81% is applied to the biomass calculated with the allometric equations for basal diameter or total height, it is sufficient to know the carbon content in a tree, a stand, or a plantation of *Pinus hartwegii* Lindl seedlings; therefore, its use is reliable." However, the adjusted model is of the where the dependent variable (Y) is biomass or carbon (in kilograms) and the independent variable is the normal diameter at 1.3 m height (DN) in centimeters determine that "the trees of this species concentrate most of the aerial carbon in the stem, followed by the branches and finally the foliage".

Based on the above, this research offers valuable information on the growth and yield of *Pinus hartwegii* forest in the states of Mexico and Puebla, in response to different environmental factors through the development of allometric equations. As well as the evaluation of carbon sequestration, which is key to understand the dynamics of the forest ecosystem facing climate change. The findings of this study are consistent with previous research that has highlighted the importance of understanding forest responses to environmental changes (Valderrama-Bonnet, 2009); the need for validated dynamic stochastic models to estimate allometric equations assessing carbon sequestration (Vargas-Larreta *et al.*, 2018; SEMARNAT, 2023); and the application of the principles of parsimony and simplicity in the selection of a model (Vanclay, 1994; Weiskittel *et al.*, 2011; Burkhart & Tomé, 2012; Barrera Pérez 2018; Vargas-Larreta *et al.* 2018).

However, this study has some limitations, because it was focused in the states of Mexico and Puebla; therefore, the results may not be generalized to other types of forests. In addition, the network of sampling sites in Mexico is not yet sufficient to draw complete conclusions about the growth of many species in stands under different treatments and at sites with different productivity (Corral *et al.*, 2014; Aguirre-Calderón, 2015; Vargas-Larreta *et al.* 2018). Finally, forestry research in Mexico still faces challenges in terms of data availability and development of models that are in fact widely used (Vargas-Larreta *et al.* 2018; SEMARNAT, 2023), thus providing a solid basis for future research to develop forest management strategies based on mathematical evidence.

CONCLUSIONS

The research achieved significant results using these statistical methods and data sources: SiBiFor databases and Power data from NASA methodology. Statistical models: Ordinary Least Squares (OLS), Ridge Model with regression, and Random Forest in R. Validation techniques: analysis of variance, F distribution theorem, residual validation graphs, statistical criterion R^2. Simulations: linear allometric equations estimated by OLS with Ridge Model, and incorporation of significant climatic variables.

It was accepted that linear regression models were significant and explained the variability of Y; and allometric equations of "commercial volumetric dimension" for *Pinus hartwegii* Lindl with significant climatic variables were estimated. The best precision in estimating the total volume of trees with bark was found with Newton traditional mathematical equations and estimated allometric equations in the states of Mexico and Puebla. The second better estimator were theoretical models (volume by timber unit, logs) for excurrent dendrometric types.

The allometric equation estimated by mathematical models with the best estimation of the TAV cc is theoretical models for excurrent dendrometric types (log volume) in the states of Mexico and Puebla with an evaluation of 90.5% and 95%, respectively. Thus, we obtained a better volumetric approach facing climate change with the Newton mathematical equations, and theoretical models for excurrent dendrometric types (volume per timber unit, logs).

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